Making Error Correcting Codes Work for Flash Memory

Part II: Algebraic and Graph-based Codes with Applications to Flash Memory

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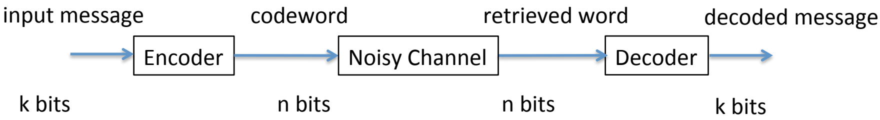
Flash Memory Summit 2013

Outline

Concepts of interest



- A channel code C maps a message m of length k into a codeword c of length n, with $n > k$ (encoder)
- Total number of codewords: 2^k
- Code rate: $R = k/n$.
- Structure of C is used to determine the stored message (decoder).

Concepts of interest

Linear block code C of dimension k and codeword length n can be represented by

- a $k \times n$ generator matrix G
- a $(n-k) \times n$ parity check matrix H

- G specifies the range space of C and H specifies the null space of C.
- The two representations are mathematically equivalent.

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Concepts of interest

Linear block codes can be divided in two categories:

- algebraic codes (BCH codes, Hamming codes, Reed-Solomon codes)
- graph-based codes (LDPC codes, Turbo codes)

A good practical channel code should

- be able to correct as many transmission errors as possible
- be equipped with a simple decoding algorithm

Algebraic Codes

Brief review of finite fields

Suppose q is prime.

- $GF(q)$ can be viewed as the set $\{0, 1, \ldots, q-1\}$.
- Operations are performed modulo q.

Example:

- $GF(5)$ has elements $\{0, 1, 2, 3, 4\}$ such that

product	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

sum	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Brief review of finite fields

- $GF(q)$ can also be expressed as $\{\alpha^{-\infty} = 0, \alpha^0 = 1, \alpha, \alpha^2, \ldots, \alpha^{q-1}\}$, for suitably chosen α.

Example:

- In $GF(5)$: $0 \to \alpha^{-\infty}$, $1 \to \alpha^0$, $2 \to \alpha$, $3 \to \alpha^3$ and $4 \to \alpha^2$

- Consider an element α of $GF(q)$ such that $\alpha \neq 0$ and $\alpha \neq 1$.
- Let s be the smallest positive integer such that $\alpha^s = 1$. Then, s is the order of α.
- If $s = q - 1$, then α is called a primitive element of $GF(q)$.

$\boxed{GF(q) \text{ is thus generated by powers of a primitive element } \alpha.}$

Brief review of finite fields

- We are often interested in the extension field $GF(q^m)$ of $GF(q)$, where q is prime and m is a positive integer.
- $GF(q^m)$ is then $\{\alpha^{-\infty} = 0, \alpha^0 = 1, \alpha, \alpha^2, \ldots, \alpha^{q^m-1}\}$, where α denotes a primitive element of $GF(q^m)$ and is a root of so-called primitive polynomial.

Example:

- $GF(8) = GF(2^3)$.
- Here, α is a root of the polynomial $x^3 + x + 1$.
- We then have

$$\begin{aligned} \alpha^0 &= 1 \\ \alpha^1 &= \alpha \\ \alpha^2 &= \alpha^2 \\ \alpha^3 &= \alpha + 1 \end{aligned} \qquad \begin{aligned} \alpha^4 &= \alpha^2 + \alpha \\ \alpha^5 &= \alpha^2 + \alpha + 1 \\ \alpha^6 &= \alpha^2 + 1 \\ \alpha^{-\infty} &= 0 \end{aligned}$$

BCH code construction

BCH code C is a linear, cyclic code described by a $(d-1)\times n$ parity check matrix H with elements from $GF(q^m)$ with α having order n:

$$H = \begin{bmatrix} 1 & \alpha^{b} & \alpha^{2b} & \cdots & \alpha^{(n-1)b} \\ 1 & \alpha^{b+1} & \alpha^{2(b+1)} & \cdots & \alpha^{(n-1)(b+1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \alpha^{b+d-2} & \alpha^{2(b+d-2)} & \cdots & \alpha^{(n-1)(b+d-2)} \end{bmatrix}$$

- b is any (positive) integer and d is integer $2 \leq d \leq n$.
- Minimum distance of C is at least d. The code corrects at least $t = \lfloor \frac{d-1}{2} \rfloor$ errors.

BCH code construction

- If α is a primitive element, then the blocklength is $n = q^m - 1$ (largest possible).
- If $b = 1$, BCH code is called narrow-sense (simplifies some encoding and decoding operations).
- For $m = 1$, BCH codes are also known as Reed-Solomon codes.

BCH code properties

- A code C is called a cyclic code if all cyclic shifts of a codeword in C are also codewords.

Example:

- Suppose $(0,1,0,1,1) \leftrightarrow x^3 + x + 1$ is a codeword in C. Then so are $(1,0,1,1,0)$, $(0,1,1,0,1)$, $(1,1,0,1,0)$ and $(1,0,1,0,1)$.

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- Cyclic code is generated by a generator polynomial $g(x)$, such that each codeword c corresponds to a polynomial $p_c(x) = m(x)g(x)$. All rows of the generator matrix G are cyclic shifts of $g(x)$.
- BCH code: Each codeword c corresponds to a polynomial $p_c(x) = m(x)g(x)$ where $g(x)$ is LCM of $(x-\alpha^b)(x-\alpha^{b+1})\cdots(x-\alpha^{b+d-2})$.

BCH code example

- Let's construct a narrow-sense BCH code over $GF(8)$ correcting $t = 1$ error and of length $n = 7$.
- We consider a primitive element α that satisfies $\alpha^3 + \alpha + 1 = 0$. Notice that $\alpha^7 = 1$.
- Then,

$$H = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 & \alpha^{10} & \alpha^{12} \end{bmatrix}$$

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BCH code example

We can interpret this code in the binary domain by substituting

$$1 \to \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \alpha \to \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \alpha^2 \to \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \alpha^3 \to \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\alpha^4 \to \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \alpha^5 \to \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \alpha^6 \to \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad 0 \to \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

BCH code example

We can then interpret this parity check matrix in the binary domain as

$$H = \left[\begin{array}{ccccccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ \hline 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Here H is 6×7 and has rank 3. This code can correct 1 error.

Decoding BCH codes

Decoding algorithm heavily relies on the algebraic structure of the code: recall that each codeword polynomial $c(x)$ must have as roots α^b, α^{b+1},..,α^{b+d-2}.

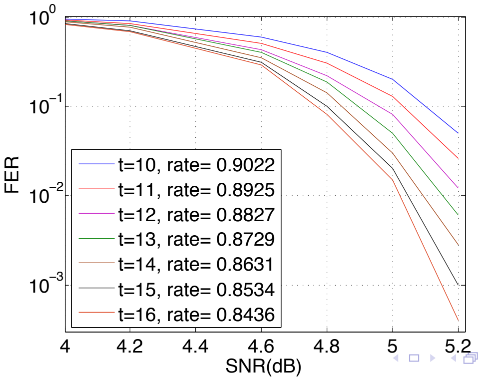
1. Compute the syndromes of the received polynomial $r(x)$– tells us which of α's are not the roots.
2. Based on the syndromes, compute the locations of the errors (system of linear equations).
3. Compute the error values at these location (system of non-linear equations that are in the Vandermode form)
4. Based on steps 2 and 3, build error polynomial $e(x)$.
5. Add $e(x)$ to $r(x)$ to produce the estimate of $c(x)$.

Decoding BCH codes

- If the system of equations cannot be solved, declare a decoding failure. This is a hard limit on the number of correctable errors.
- Implementation can be greatly reduced using the shift-registers viewpoint in the Berlekamp-Massey algorithm.

Performance evaluation

Figure: Theoretical bound for length $n = 1023$ binary BCH code for different error correction capability t (and different code rate).



Graph-Based Codes

Low Density Parity Check (LDPC) Codes

Definition 1: LDPC code

An LDPC block code C is a linear block code whose parity-check matrix H has a small number of ones in each row and column.

- Invented by Gallager in 1963 but were all but forgotten until late 1990's.
- In the limit of very large block-lengths LDPC codes are known to approach the Shannon limit (i.e., the highest rate at which the code can be designed that guarantees reliable communication)
- LDPC codes are amenable to low-complexity iterative decoding.

An Example

LDPC code described by the sparse parity check matrix H:

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Matrix H has 9 columns and 6 rows.

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Matrix H has 9 columns and 6 rows.
There are 9 coded bits and 6 parity-check equations.
Each coded bit participates $\ell=2$ parity-check equations and each parity-check equation contains $r = 3$ coded bits.

LDPC Preliminaries

Definition 3: Tanner graph

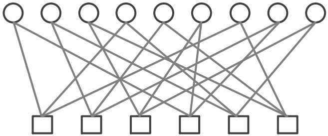
A Tanner graph of a code C with a parity check matrix H is the bipartite graph such that:

- each coded symbol i is represented by a variable node v_i,
- each parity-check equation j is represented by a check node c_j,
- there exists an edge between a variable node and a check node if and only if $H(j, i) = 1$.

An Example

LDPC code: parity check matrix H and its Tanner graph

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$



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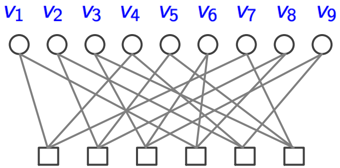
LDPC code: parity check matrix H and its Tanner graph

$$\begin{array}{c} H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \\ \quad\;\; v_1 \;\; v_2 \;\; v_3 \;\; v_4 \;\; v_5 \;\; v_6 \;\; v_7 \;\; v_8 \;\; v_9 \end{array}$$



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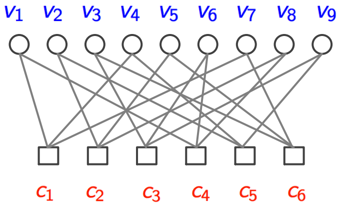

Message Passing Decoding

Message-passing (belief propagation) is an iterative decoding algorithm that operates on the Tanner graph of the code.
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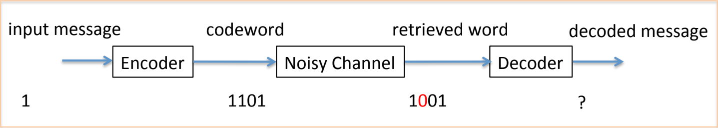
2. (check processing) Each check node then computes the consistency of incoming messages,
3. (check-to-bit) Each check node then sends a message to each variable node it is connected to,
4. (bit processing) Each variable node (coded symbol) updates its value.

Message Passing Decoding

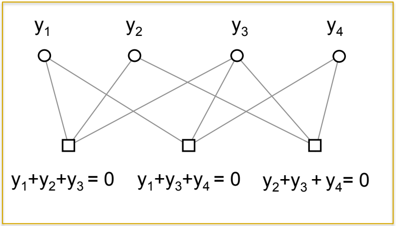
Passed messages can be either

- Hard decisions: 0 or 1
- Soft decisions/likelihoods: real numbers

An Example



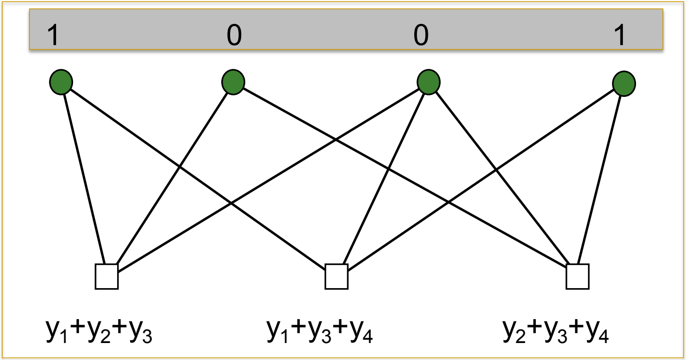
Message m		Codeword y
m_1		$y_1y_2y_3y_4$
0	$\rightarrow$	0 0 0 0
1	$\rightarrow$	1 1 0 1



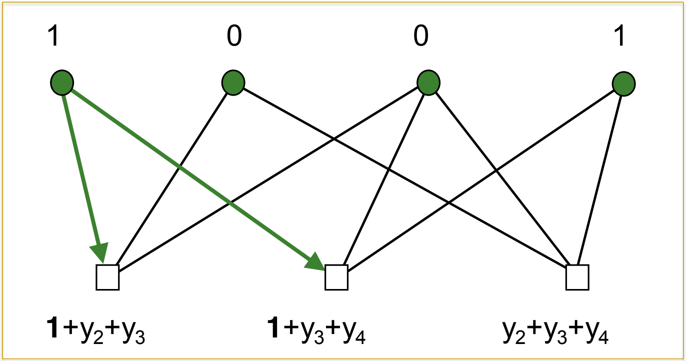
Message Passing Decoding

Bit-flipping algorithm

Received Codeword



Bit-to-Check Messages



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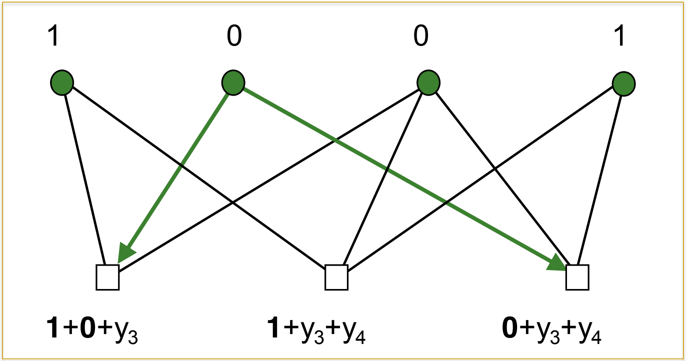
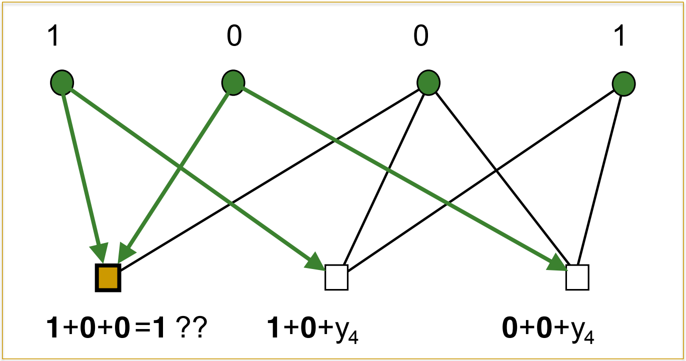


Check Processing



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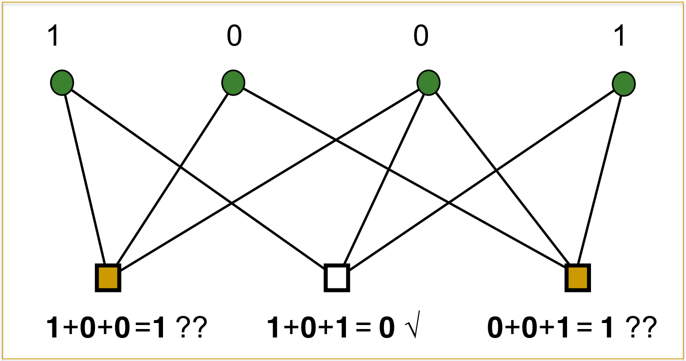
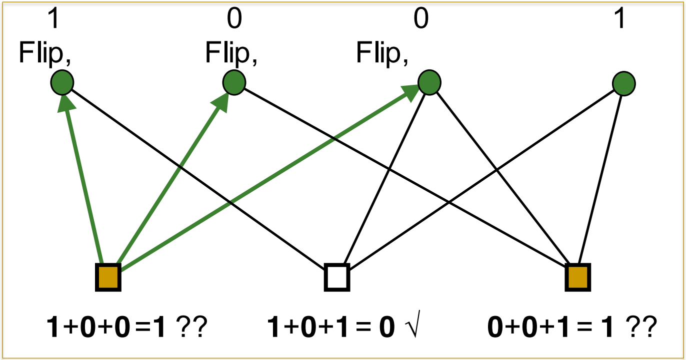


Check-to-Bit Messages



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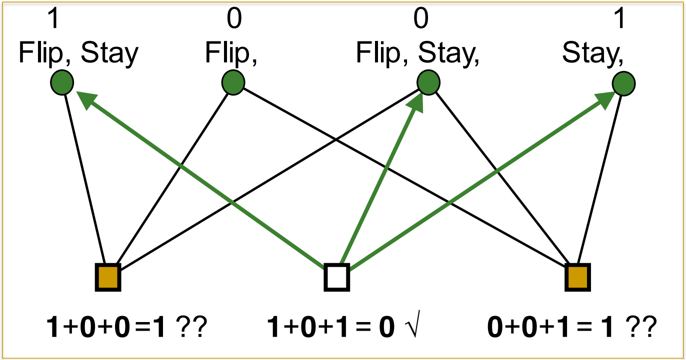

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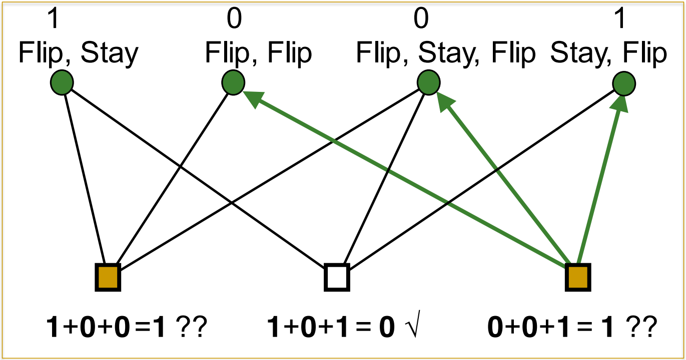

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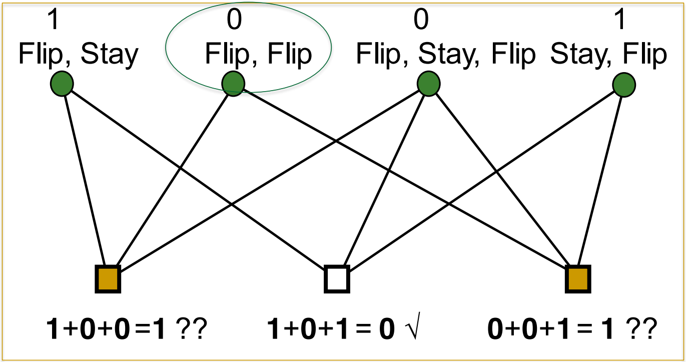

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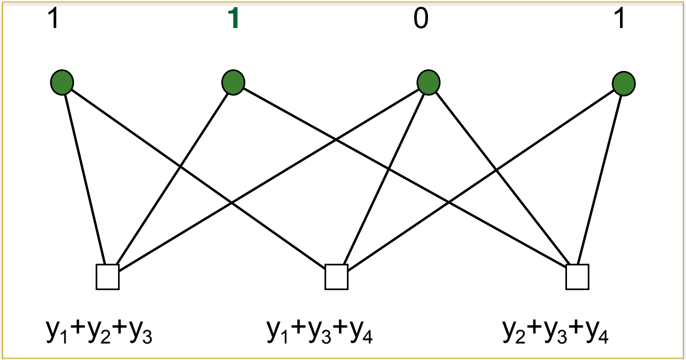

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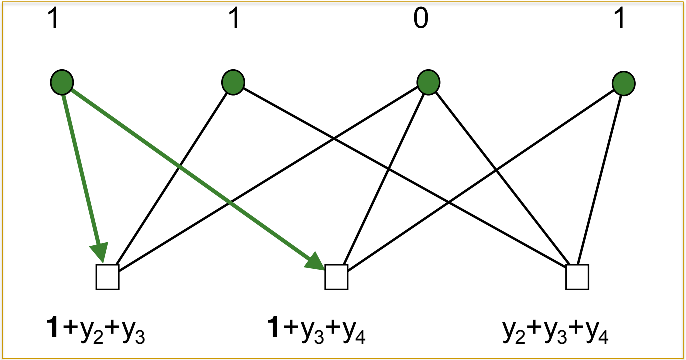

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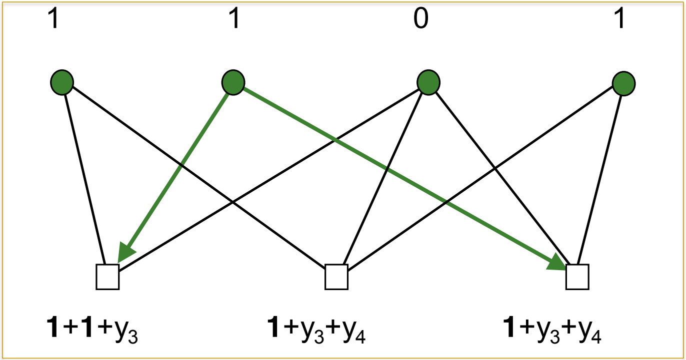

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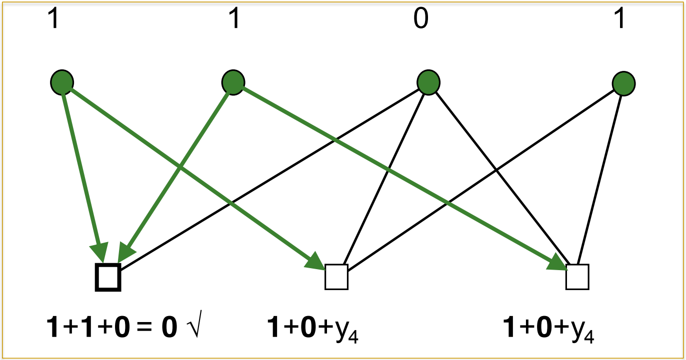

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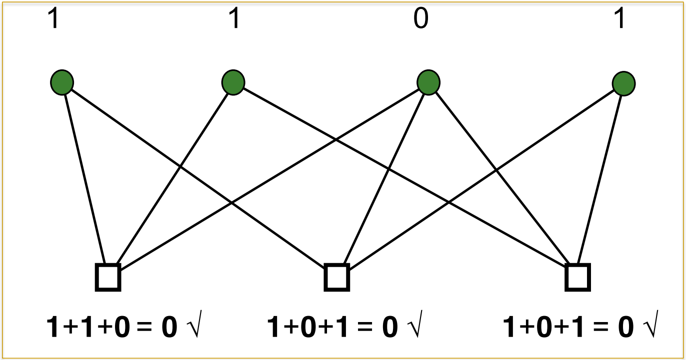

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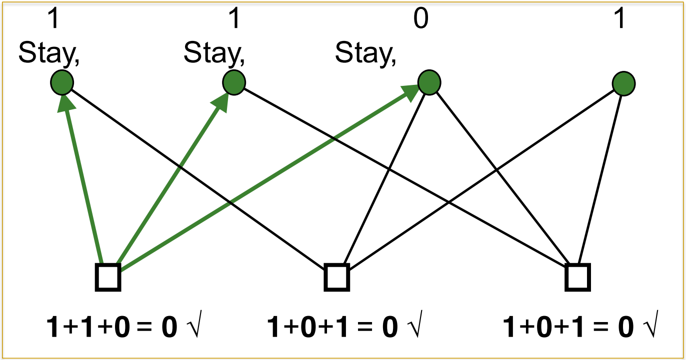

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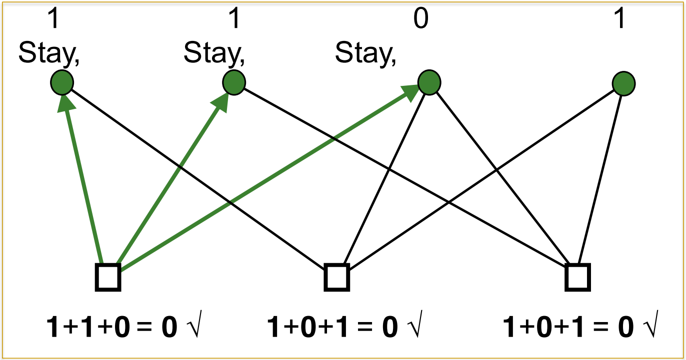

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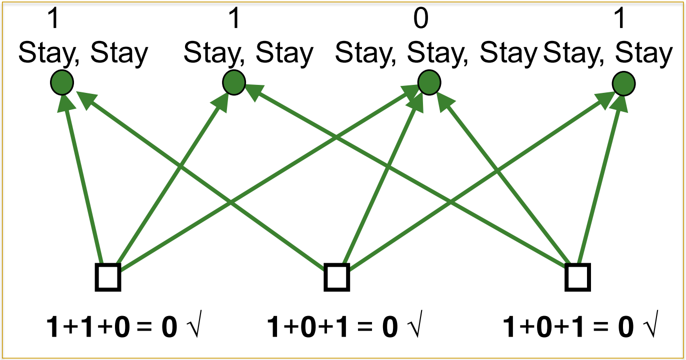

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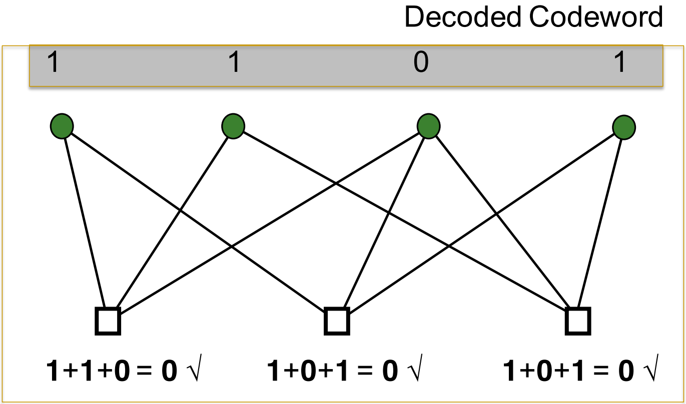

Soft Iterative Decoding

Improved variants of message passing algorithm use soft information as messages, i.e., log-likelihood ratio $L = \log \frac{P(x_i=0|y_i)}{P(x_i=1|y_i)}$.

Sum-product algorithm (SPA) [1,2]

Min-sum algorithm (MSA) [3]

[1] R. Gallager, *MIT Press*, 1963.
[2] T. Richardson and R. Urbanke, *IEEE Trans. on Info. Theory*, 2001.
[3] M. P. C. Fossorier, M. Mihaljevic, and H. Imai, *IEEE Trans. on Comm.*, 1999.

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Sum-product algorithm (SPA) [1,2]

- bit-to-check $L(v_i \rightarrow c_j) = \sum_{j' \in N(i)\backslash j} L(c_j' \rightarrow v_i) + L^{int}(v_i)$
- check-to-bit $L(c_j \rightarrow v_i) = \Phi^{-1}\left(\sum_{i' \in N(j)\backslash i} \Phi(|L(v_i' \rightarrow c_j)|) \sum_{i' \in N(j)\backslash i} sgn(L(v_i' \rightarrow c_j))\right)$ where $\Phi(x) = -\log(\tanh(x/2))$

Min-sum algorithm (MSA) [3]

- check-to-bit $L(c_j \rightarrow v_i) = \min_{i' \in N(j)\backslash i} |L(v_i' \rightarrow c_j)| \prod_{i' \in N(j)\backslash i} sgn(L(v_i' \rightarrow c_j))$

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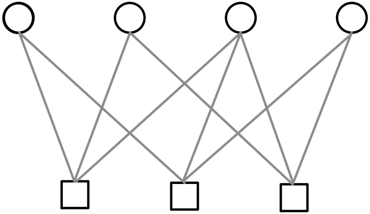
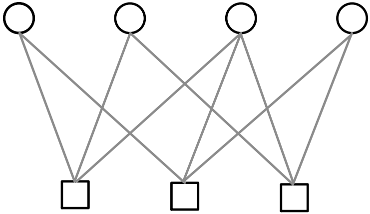
Soft Decoding

Bit values 1 1 0 1



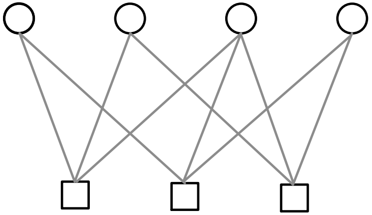
Soft Decoding

Bit values	1	1	0	1
Values using BPSK	-1	-1	+1	-1



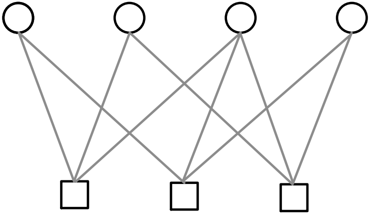
Soft Decoding

Bit values	1	1	0	1
Values using BPSK	-1	-1	+1	-1
Values from channel	-1.1	0.1	1.2	-0.9



Soft Decoding

Bit values	1	1	0	1
Values using BPSK	-1	-1	+1	-1
Values from channel	-1.1	0.1	1.2	-0.9
Beliefs ($L_{v_i}^{(int)}$)	-2.2	0.2	2.4	-1.8



$$L_{v_i}^{(int)} = log\left(\frac{e^{-(y_i-1)^2/2\sigma_n^2}}{e^{-(y_i+1)^2/2\sigma_n^2}}\right) = \frac{2}{\sigma_n^2} y_i$$

We assume $\sigma_n = 1$.

Soft Decoding

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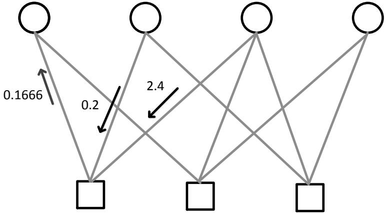

$$L_{c_j \to v_j} = 2\tanh^{-1}\left(\prod_{\substack{l \neq i \\ v_l \to c_j}} \tanh \frac{1}{2} L_{v_l \to c_j}\right)$$

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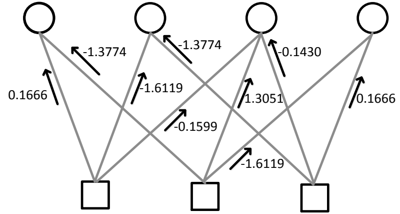

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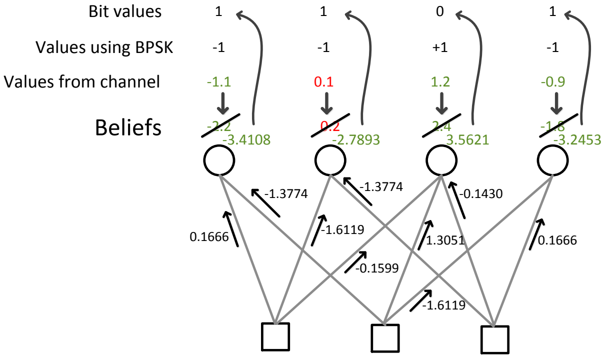
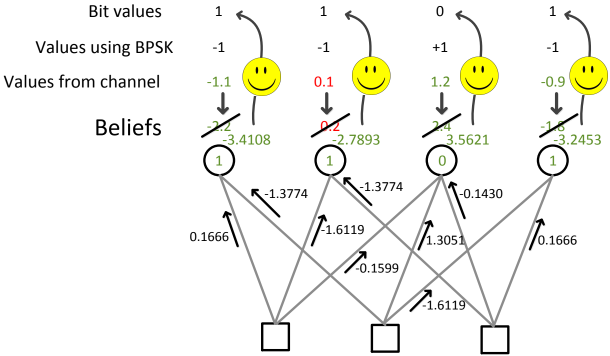
Soft Decoding



$$L_{v_i} = L_{v_i}^{(int)} + \sum_{c_j \to v_i} L_{c_j \to v_j}$$

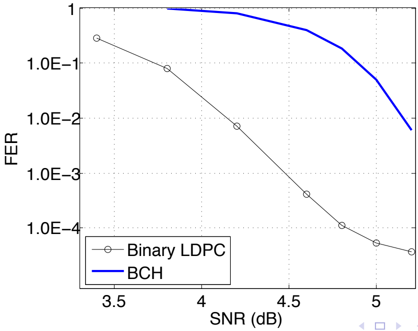
Soft Decoding



All variable nodes are decoded to correct bit value.

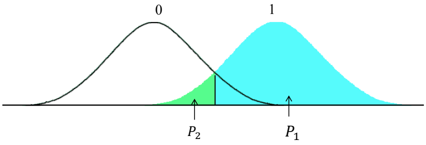
Performance evaluation

Figure: Bnary LDPC codes vs. BCH codes performance comparison for AWGN channel. Code rate is 0.9, block length is 1000 bits. BCH code corrects 13 errors.



Extracting soft information in SLC Flash

- In Flash, levels are represented by distributions
- 1 read compares against a single threshold

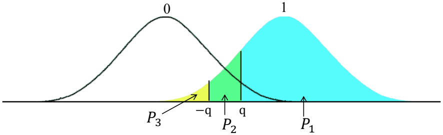


Extracting soft information in SLC Flash

- Idea: multiple word line reads

Extracting soft information in SLC Flash

- Idea: multiple word line reads
- 2 reads compare against two thresholds



Extracting soft information in SLC Flash

- Idea: multiple word line reads
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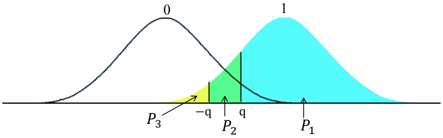

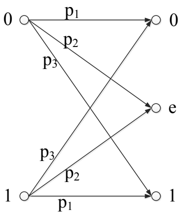

Extracting soft information in SLC Flash

- Idea: multiple word line reads
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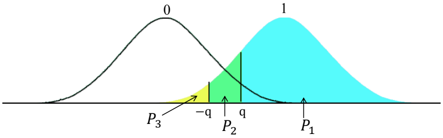
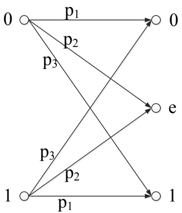




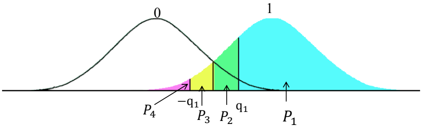
- Maximize mutual information of the induced channel to determine the best thresholds (here q and $-q$)

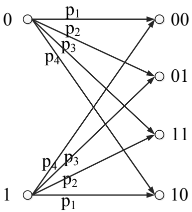
Extracting soft information in SLC Flash

- Idea: multiple word line reads

Extracting soft information in SLC Flash

- Idea: multiple word line reads
- 3 reads compare against three thresholds

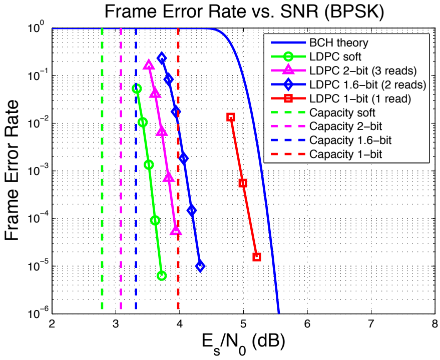




- Maximize mutual information of the induced channel to determine the best thresholds (here q_1, $-q_1$ and 0)

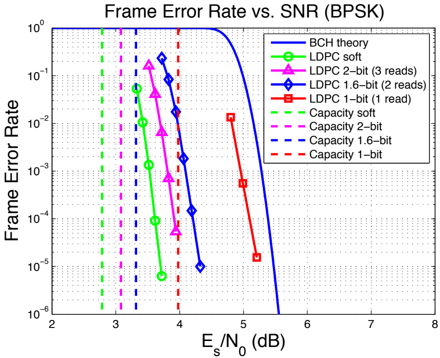
LDPC vs. BCH code performance

Figure: Performance comparison for 0.9-rate LDPC and BCH codes of length $n = 9100$.



LDPC vs. BCH code performance

Figure: Performance comparison for 0.9-rate LDPC and BCH codes of length $n = 9100$.

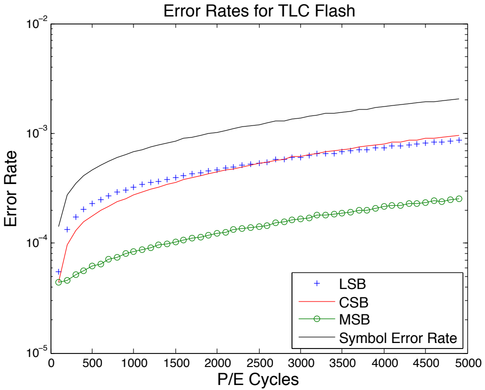


- Caution: AWGN-optimized LDPC codes may not be the best for the quantized Flash channel !

Advanced Coding Approaches

Graded algebraic codes

Motivation: Raw error rate for TLC flash



LSB: least significant bit
CSB: center significant bit
MSB: most significant bit

Table: Mapping between Voltage Levels and Triple-bit Words

Voltage Level	Triple-bit Word
0	111
1	110
2	100
3	101
4	001
5	000
6	010
7	011

Error patterns within a TLC cell

Number of bits in symbol that err	Percentage of errors
1	0.9617
2	0.0314
3	0.0069

- Standard error-correction codes are designed to correct all symbol–to–symbol errors and do not differentiate among these errors.

Error patterns within a TLC cell

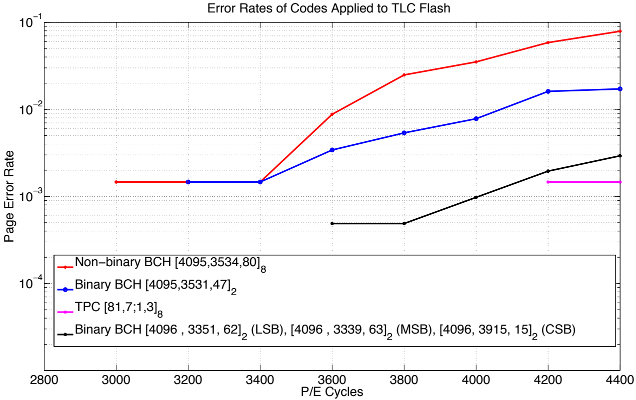
Number of bits in symbol that err	Percentage of errors
1	0.9617
2	0.0314
3	0.0069

- Standard error-correction codes are designed to correct all symbol–to–symbol errors and do not differentiate among these errors.
- Usage of standard codes: overkill in terms of redundancy, as certain symbol–to–symbol errors are not as important.

Graded algebraic codes

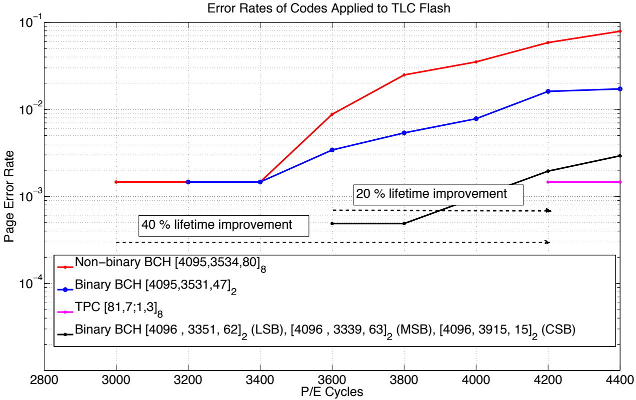
- Idea: Design codes for the observed intracell error patterns
- Approach: Algebraic codes that simultaneously control the number of symbols in error and the number of bits in error per erroneous symbol
- Construction: Tensor-product operations

Performance evaluation



All codes are of rate 0.86 and length 4000 bits.

Performance evaluation

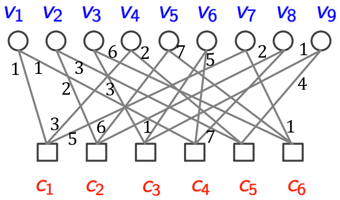


All codes are of rate 0.86 and length 4000 bits.

Non-binary LDPC codes

Entries in the parity check matrix H are taken from $GF(q)$.
Example: $GF(8) = 0, 1, 2, ..., 7$. (with $\alpha^k \to k+1$ for $0 \leq k \leq 6$)

$$\begin{array}{cc} H = \begin{bmatrix} 1 & 0 & 0 & 3 & 0 & 0 & 5 & 0 & 0 \\ 0 & 2 & 0 & 0 & 6 & 0 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 5 & 0 & 7 & 0 \\ 0 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 6 & 0 & 7 & 0 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{array} \\ \begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} & \end{array}$$



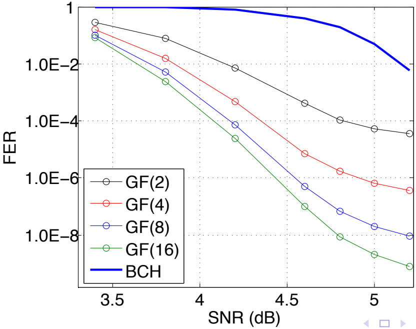
Parity check c_3: $3v_3 + v_6 + v_9 \equiv 0 \mod 8$.

Non-binary LDPC codes

- Decoding is more complex than in the binary case. Keep track of $q-1$ likelihoods on each edge.
- Popular approaches:
  - Direct implementation has complexity on the order of $O(q^2)$
  - FFT-based SPA has complexity on the order of $O(q \log q)$
  - Min-sum and its variants can further reduce the complexity

Performance evaluation

Figure: Non-binary LDPC codes vs. BCH codes performance comparison for AWGN channel. Code rate is 0.9, block length is 1000 bits. BCH code corrects 13 errors.



Algebraic codes (BCH)

- − Performance is acceptable
- + Guaranteed error correction capability
- + Structure allows for efficient decoder implementation
- − Not amenable for soft decoding

Graph-based codes (LDPC)

- + Performance is excellent
- − No guaranteed error correction capability (but we have ideas)
- − Decoder complexity is acceptable but now low
- + Amenable for soft decoding

With the move to MLC/TLC technologies, advanced coding schemes will need to be considered!

Further information, papers, references etc. available at http://loris.ee.ucla.edu

Selected list:

- R. Gabrys, E. Yaakobi and L. Dolecek, "Graded bit error correcting codes with applications to Flash memory," *IEEE Transactions on Information Theory*, vol. 59(4), pp. 2315 – 2327, Apr. 2013.
- J. Wang, L. Dolecek and R. Wesel, "The Cycle Consistency Matrix Approach to Absorbing Sets in Separable Circulant-Based LDPC Codes," IEEE Transactions on Information Theory, vol. 59(4), pp. 2293 – 2314, Apr. 2013.
- B. Amiri, J. Kliewer, and L. Dolecek, "Analysis and Enumeration of Absorbing Sets for Non-Binary Graph-Based Codes," submitted to IEEE Transactions on Communications, 2013. (Conference version in ISIT 2013.)

UCLA Coding talks and posters at 2013 Flash Summit

- R. Gabrys, "Coding for Unreliable Flash Memory Cells," Session 301-A: Flash Controller Design Options - from 8:30 to 9:40 am on Thursday, August 15.
- B. Amiri, "Low Error Floor LDPC Codes and Their Practical Decoders for Flash Memory Applications," Hall B, booths 916-920 – Exhibit Hours
- K. Vakilinia, "Non-Binary LDPC Code Design from Inter-Connected Cycles," Hall B, booths 916-920 – Exhibit Hours

Announcement

New center on Coding for Storage at UCLA:
http://www.loris.ee.ucla.edu/codess

Kick-off day on Thursday 9/19/2013!

Registration is free. Register early, space is limited.