



# Joint Rewriting and Error Correction in Flash Memories

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joint work with

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# The Problem of Block Erasure

- Erasing a cell requires block erasure
  - Block erasure slows down programming
  - Degrades flash cells
  
- Solutions
  - Flash translation layer (FTL)
  - Coding for rewriting data
    - Floating codes
    - Buffer codes
    - Rank modulation codes
    - Write-once memory (WOM) codes



## What is a “rewrite” ?

- Store data by programming cells which have been programmed before without erasing a cell.
- Constraint: *a cell previously at level 1 (high threshold voltage) has to stay at level 1 after rewriting.*
  - e.g. 101 -> 111
- After rewriting, the data stored by the previous writes no longer need to be recovered.

# The first write-once memory (WOM) code

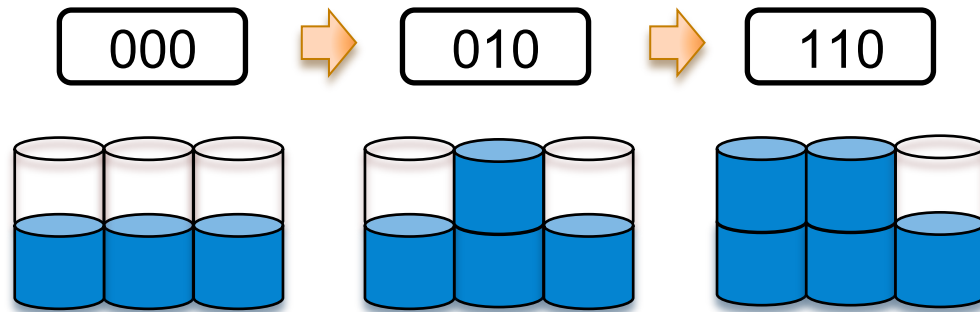
R. L. Rivest and A. Shamir, "How to reuse a 'Write-Once' memory," Information and Control, vol. 55, pp. 1–19, 1982 (also published in STOC)

Data	Codeword (1 <sup>st</sup> write)	Codeword (2 <sup>nd</sup> write)
00	000	111
01	001	110
10	010	101
11	100	011

Write 2 bits twice using 3 cells

Rate =  $4/3 = 1.33$  bits/cell

Example (with SLC): we first write data 10, then rewrite the data to 01.



Capacities have been derived

[1] **C. Heegard**, “On the capacity of permanent memory”, IEEE Transactions on Information theory, vol. 31, no. 1, 1985

Different WOM codes have been proposed.

Capacity-achieving codes have been proposed

[1] **A. Shipilka**, “Capacity achieving multiwrite WOM codes”, 2012.

[2] **D. Burshtein and A. Strugatski**, “*Polar write-once memory codes*,” ISIT 2012.

However, WOM codes for noisy channels are limited.

[3] **G. Zemor and G. D. Cohen**, “Error-Correcting WOM-Codes”, IEEE Transactions on Information Theory, vol. 37, no. 3, pp. 730-734, 1991.

[4] **E. Yaakobi, P. Siegel, A. Vardy, and J. Wolf**, “Multiple Error-Correcting WOM-Codes”, in IEEE Transactions on Information Theory, vol. 58, no. 4, pp. 2220-2230, 2012.

We study WOM codes which correct many errors.



## Polar WOM Codes<sup>[1]</sup>

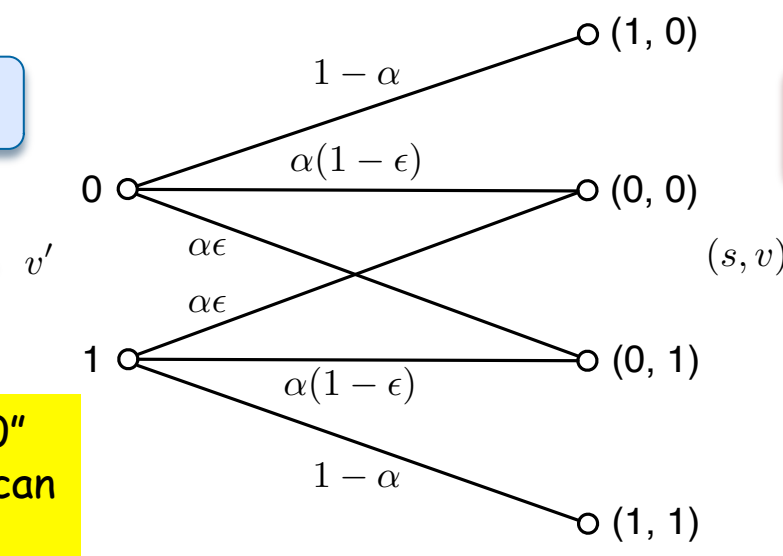
- Views a write as the decoding of a polar code:
  - Views the cells' state before the write as a noisy Polar codeword.
  - Views the cells' state after the write as the corrected (i.e., error-free) Polar codeword.
- More precisely, write/rewrite can be considered as lossy data compression.

[1] D. Burshtein and A. Strugatski, "Polar write-once memory codes," ISIT 2012.

# The Channel for Rewriting

- Smart idea by Burshtein and Strugatski:
  - Add dither to cell levels:
    - Let  $s \in \{0, 1\}$  be the level of a cell.
    - Let  $g \in \{0, 1\}$  be a pseudo-random number known to the WOM encoder and the WOM decoder.
    - Let  $v = s \oplus g$  be called the value of the cell.
  - Build a test channel for the write, which we shall call the WOM channel

Cell value after rewrite



Cell states and values before rewrite

$\alpha$ : fraction of the cells at "0"  
 $\epsilon$ : fraction of the cells you can program



# Model for

# WOM codes



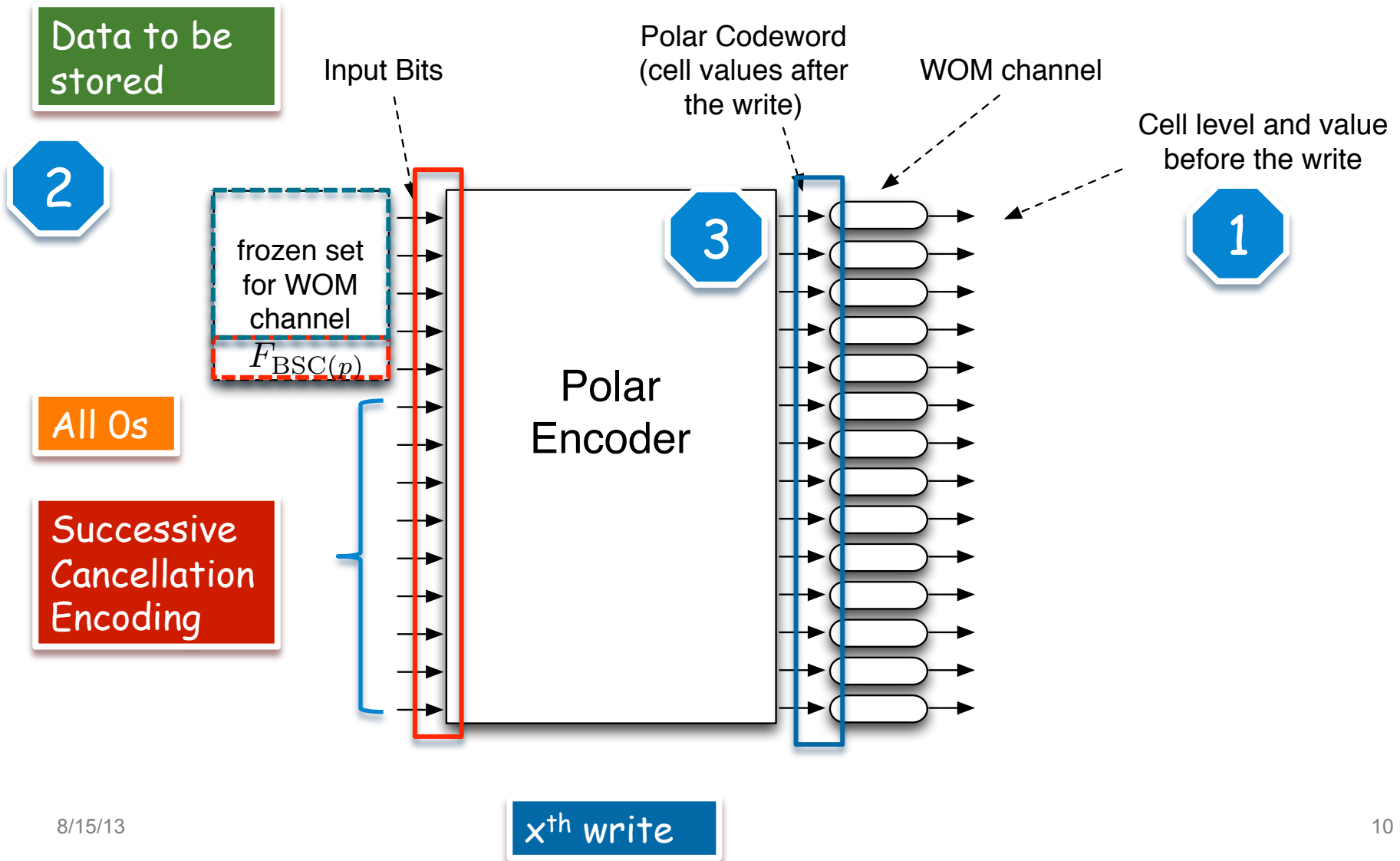


## ECC WOM codes

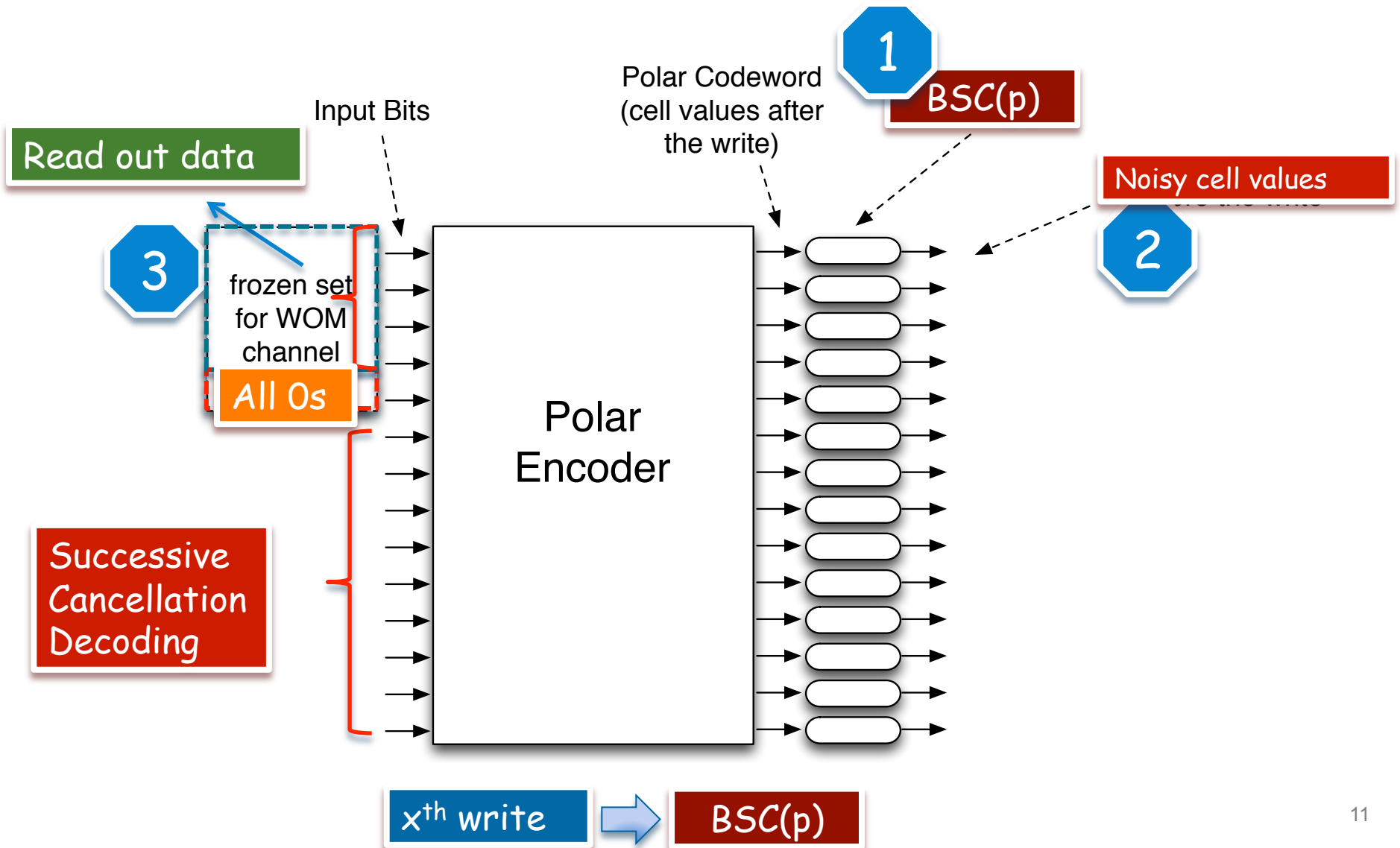
- We would like to construct a nested code.
  - A WOM codeword is also a channel codeword
- Consider two channels
  - WOM channel. Let its frozen set be  $F_{\text{WOM}}(\alpha, \epsilon)$
  - BSC/noise channel. Let its frozen set be  $F_{\text{BSC}}(p)$
- A codeword of polar codes for WOM channel is also a codeword of the codes for BSC channel under the condition

$$F_{\text{BSC}}(p) \subseteq F_{\text{WOM}}(\alpha, \epsilon)$$

# The Encoding Scheme

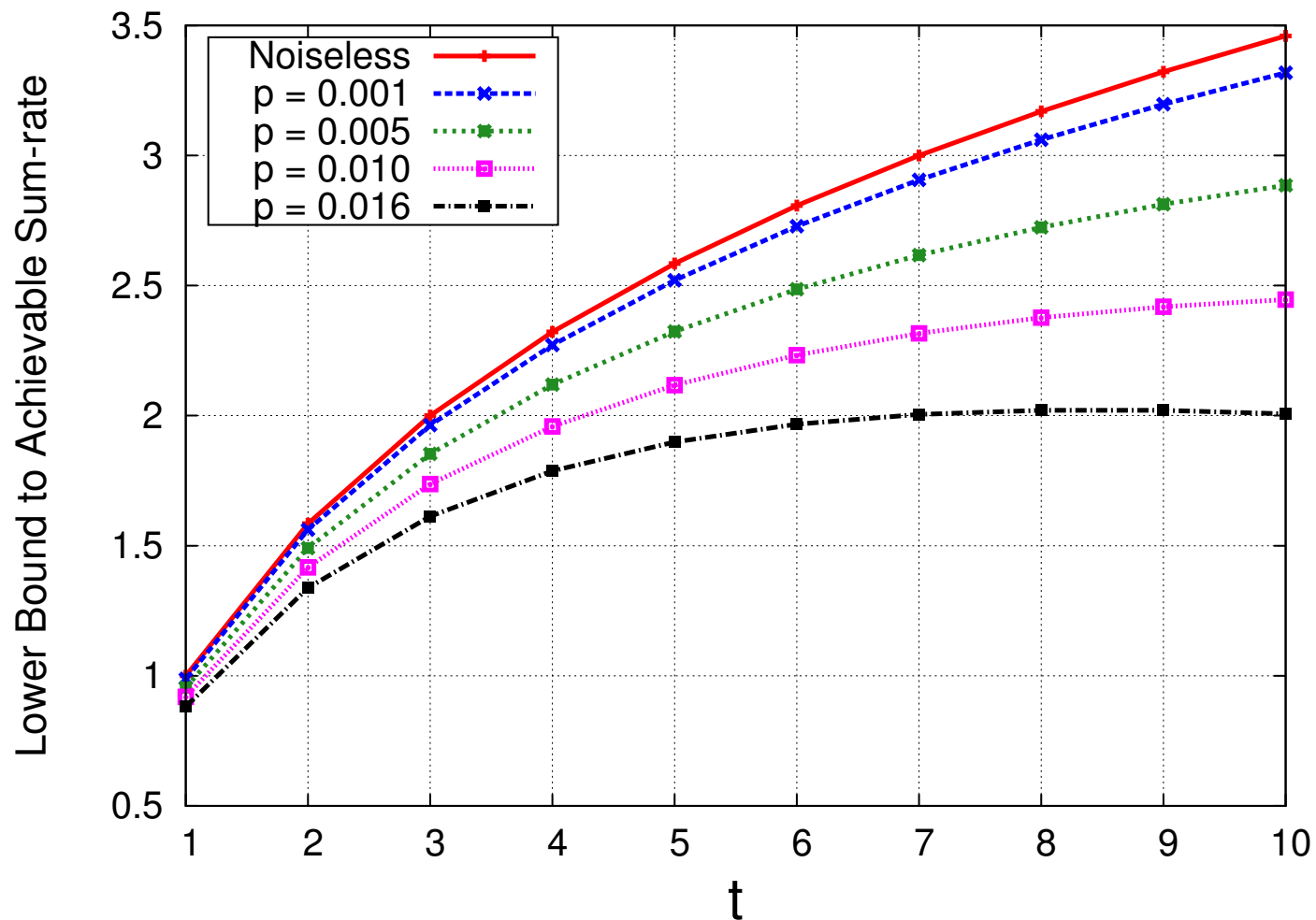


# The Decoding Scheme



# Lower Bounds to Achievable Sum-Rates

Sum-rate: total number of bits that can be stored using one cell through  $t$  writes.



## Conclusions

- We proposed a coding scheme
  - allows multiple rewrites in one P/E cycle.
  - corrects a significant number of errors
  - Uses polar lossy source coding and channel coding

**A. Jiang, Y. Li, E. En Gad, M. Langberg and J. Bruck.** Joint Rewriting and Error Correction in Write-Once Memories. ISIT 2013.



THANK YOU