Erasure Codes Made So Simple, You'll Really Like Them

W. David Schwaderer
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schwaderer_01@comcast.net

Agenda

- Errors Versus Erasures
- HDD Bit Error Rate Implications
- RAID 4, 5, and 6 Review
- Objects and Dispersed Storage
- The Math
- Summary
- Questions

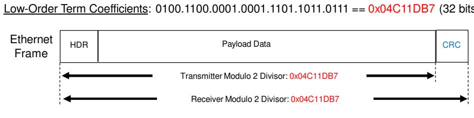
Errors Versus Erasures

Data Communications *Error Detection*

- Asynchronous (Start/Stop) Communication – Parity (e.g. Even/Odd)
- TCP/IP Checksums
- Ethernet CCITT-32 CRC

(x+1) * *Special* Polynomial: $(x^{32}+x^{26}+x^{23}+x^{22}+x^{16}+x^{12}+x^{11}+x^{10}+x^{8}+x^{7}+x^{5}+x^{4}+x^{2}+x^{1}+1)$

Low-Order Term Coefficients: 0100.1100.0001.0001.1101.1011.0111 == 0x04C11DB7 (32 bits)



Erasure: Error in known position

Key Take Away:

Special Polynomials; Magic Numbers; Bit-Wise, Modulo 2 Arithmetic

HDD Facts of Life

Nominal raw BER values: 10^{-5} to 10^{-6} => Improve reliability using ECC

ECC-corrected BER ratings:
Desktop hard disks ~ 1 in 10^{14}
Enterprise hard disks ~ 1 in 10^{15} (ten times better).

Desktop hard disks have ~8X the capacity of an enterprise disks.

=> Large capacity disks experience inevitable uncorrectable bit errors

Disk arrays use RAID to improve reliability.

(Spreads data across multiple disks)

RAID ~ Redundant Array of *Inexpensive* Disks

~ Redundant Array of *Independent* Disks

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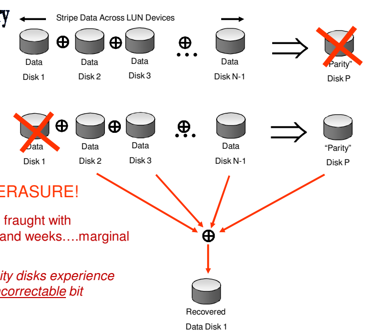
RAID 4 & 5 Parity



RAID 5 *Distributes* Parity

Rebuilds are fraught with peril….days and weeks….marginal reliability…

Large capacity disks experience inevitable uncorrectable bit errors…

"N" Total Disks, "N-1" Data Disks, "*N-1 of N*"
Can Lose One Disk, May Have Rebuild Problems

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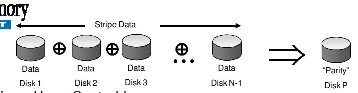
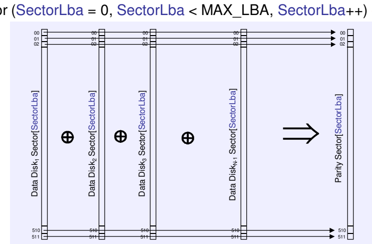
RAID 4 & 5 Parity



RAID 5 Distributes Parity

```
unsigned long SectorLba;
for (SectorLba = 0, SectorLba < MAX_LBA, SectorLba++) {
```




```
}
```

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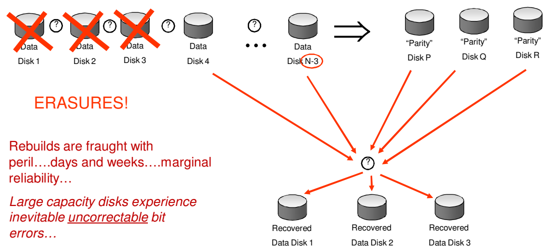
RAID 6 Parity



ERASURES!

Rebuilds are fraught with peril….days and weeks….marginal reliability…

Large capacity disks experience inevitable uncorrectable bit errors…

"N" Total Disks, "N-3" Data Disks, "*N-3 of N*"
Can Lose Three Disks, Even Fewer Rebuild Problems

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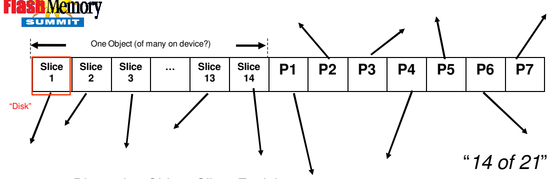
Objects & Information Dispersal



Dispersing Object *Slices* Exploits:

- Collective storage device pool isolation/reliability
- Distributed scale-out infrastructure design strengths
  1. Distributed rebuilds for lost object slices
  2. Network bandwidth load balancing

Eliminates 7X data redundancy and RAID rebuild weaknesses.

However, extreme data availability brings no free lunch:

- Computations
- Read/Update/Write Workload Amplification
- *Scale at Tail* Laggard Compensations

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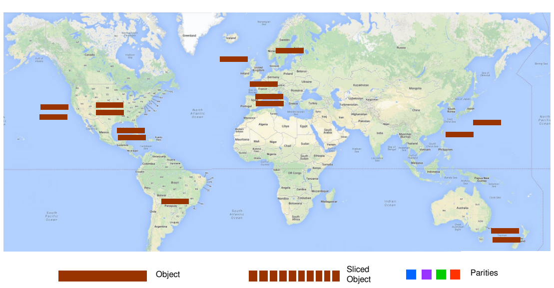
Replication Storage Reclamation



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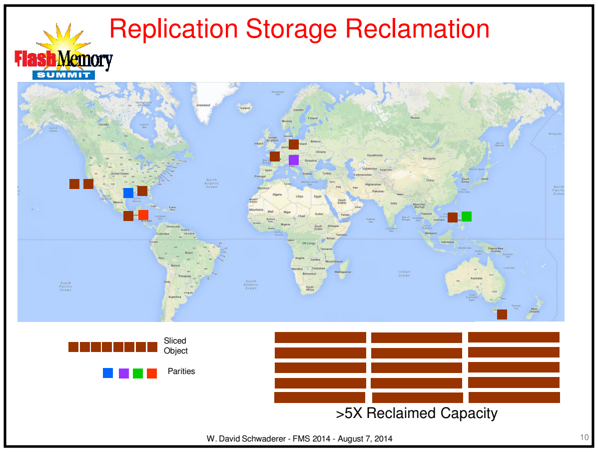
Replication Storage Reclamation
Sliced Object
Parities
>5X Reclaimed Capacity
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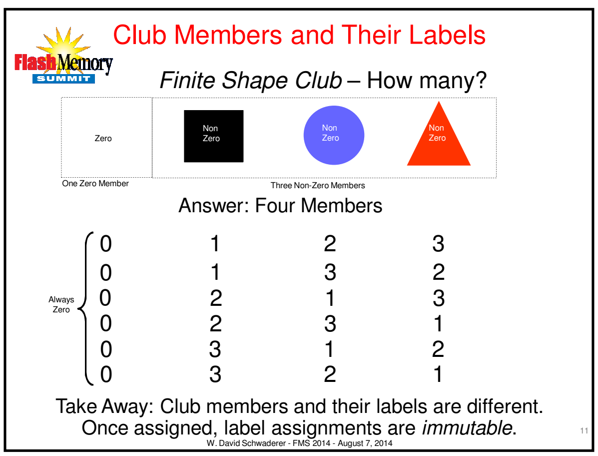
Club Members and Their Labels
Finite Shape Club – How many?
Zero
Non Zero
Non Zero
Non Zero
One Zero Member
Three Non-Zero Members
Answer: Four Members
Always Zero
0 1 2 3
0 1 3 2
0 2 1 3
0 2 3 1
0 3 1 2
0 3 2 1
Take Away: Club members and their labels are different.
Once assigned, label assignments are *immutable*.
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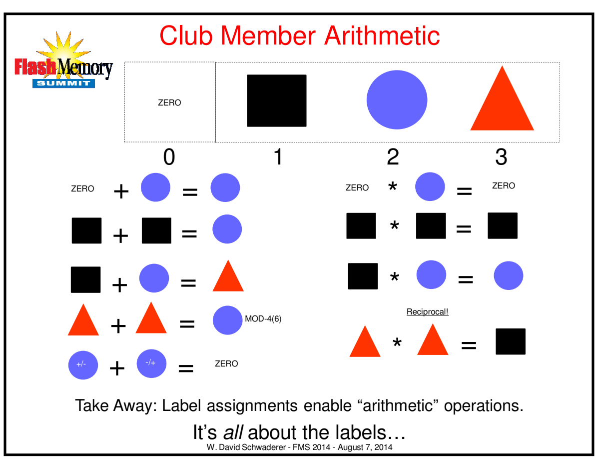
Club Member Arithmetic
ZERO
0 1 2 3
ZERO + =
+ =
+ =
+ = MOD-4(6)
+/- + -/+ = ZERO
ZERO * = ZERO
* =
* =
Reciprocal!
* =
Take Away: Label assignments enable "arithmetic" operations.
It's *all* about the labels…
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Polynomial Shorthand

Consider Degree 8 Polynomials

$aX^8 + bX^7 + cX^6 + dX^5 + eX^4 + fX^3 + gX^2 + hX^1 + kX^0$

Example Degree 8 Polynomial

$4X^8 + 0X^7 + 2X^6 + 16X^5 + 10X^4 + 8X^3 + (-22)X^2 + 19X^1 + 36X^0$

Example Degree 8 Polynomial Shorthand

4.0.2.16.10.8.-22.19.36 Label!

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$X^8 + X^4 + X^3 + X^2 + 1$ (*Special* Polynomial)

$X^8 + 0X^7 + 0X^6 + 0X^5 + 1X^4 + 1X^3 + 1X^2 + 0X^1 + 1X^0$ = 1.0001.1101
= 0x1D Polynomial *Label!*

If α is a root, by definition:

$\alpha^8 + \alpha^4 + \alpha^3 + \alpha^2 + 1 = 0$

So,

$\alpha^8 + \alpha^4 + \alpha^3 + \alpha^2 + 1 = 0$
$\oplus(\alpha^4 + \alpha^3 + \alpha^2 + 1) = \oplus(\alpha^4 + \alpha^3 + \alpha^2 + 1)$

or: $\alpha^8 = \alpha^4 + \alpha^3 + \alpha^2 + 1$

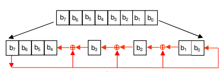
$\alpha^0 = 1 = 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 0\alpha^4 + 0\alpha^3 + 0\alpha^2 + 0\alpha + 1$ ~ 0x01
$\alpha^1 = \alpha = 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 0\alpha^4 + 0\alpha^3 + 0\alpha^2 + 1\alpha + 0$ ~ 0x02
$\alpha^2 = \alpha^2 = 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 0\alpha^4 + 0\alpha^3 + 1\alpha^2 + 0\alpha + 0$ ~ 0x04
$\alpha^3 = \alpha^3 = 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 0\alpha^4 + 1\alpha^3 + 0\alpha^2 + 0\alpha + 0$ ~ 0x08
$\alpha^4 = \alpha^4 = 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 1\alpha^4 + 0\alpha^3 + 0\alpha^2 + 0\alpha + 0$ ~ 0x10
$\alpha^5 = \alpha^5 = 0\alpha^7 + 0\alpha^6 + 1\alpha^5 + 0\alpha^4 + 0\alpha^3 + 0\alpha^2 + 0\alpha + 0$ ~ 0x20
$\alpha^6 = \alpha^6 = 0\alpha^7 + 1\alpha^6 + 0\alpha^5 + 0\alpha^4 + 0\alpha^3 + 0\alpha^2 + 0\alpha + 0$ ~ 0x40
$\alpha^7 = \alpha^7 = 1\alpha^7 + 0\alpha^6 + 0\alpha^5 + 0\alpha^4 + 0\alpha^3 + 0\alpha^2 + 0\alpha + 0$ ~ 0x80

$\alpha^8 = \alpha^4 + \alpha^3 + \alpha^2 + 1$
$= 0\alpha^7 + 0\alpha^6 + 0\alpha^5 + 1\alpha^4 + 1\alpha^3 + 1\alpha^2 + 0\alpha + 1$ ~ 0x1D

$\alpha^9 = \alpha(\alpha^8) = \alpha(\alpha^4 + \alpha^3 + \alpha^2 + 1) = \alpha^5 + \alpha^4 + \alpha^3 + \alpha$ ~ 0x3A
$\alpha^{10} = \alpha^2(\alpha^8) = \alpha^2(\alpha^4 + \alpha^3 + \alpha^2 + 1) = \alpha^6 + \alpha^5 + \alpha^4 + \alpha^2$ ~ 0x74
$\alpha^{11} = \alpha^3(\alpha^8) = \alpha^3(\alpha^4 + \alpha^3 + \alpha^2 + 1) = \alpha^7 + \alpha^6 + \alpha^5 + \alpha^3$ ~ 0xE8

$\alpha^{12} = \alpha^4(\alpha^8) = \alpha^4(\alpha^4 + \alpha^3 + \alpha^2 + 1) = \alpha^8 + \alpha^7 + \alpha^6 + \alpha^4$
$= (\alpha^4 + \alpha^3 + \alpha^2 + 1) + \alpha^7 + \alpha^6 + \alpha^4$
$= \alpha^7 + \alpha^6 + \alpha^3 + \alpha^2 + 1$ ~ 0xCD

...



Linear Feedback Shift Register

$\alpha^{254} = \alpha^7 + \alpha^3 + \alpha^2 + \alpha^1$ ~ 0x8E

$\alpha^{255} = \alpha(\alpha^{254}) = \alpha(\alpha^7 + \alpha^3 + \alpha^2 + \alpha^1) = \alpha^8 + \alpha^4 + \alpha^3 + \alpha^2$
$= (\alpha^4 + \alpha^3 + \alpha^2 + 1) + \alpha^4 + \alpha^3 + \alpha^2$
$= 1$
$= \alpha^0$

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Linear Algebra Review

X + Y + 4 + 2 = 9
3X - Y + 6 - 5 = 2

X + Y = 3
\+ 3X - Y = 1

4X = 4

=> X == 1
X + Y = 3 => Y == 2

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Galois Field Overview

A <u>finite</u> set of elements (e.g. 2^8, 2^{16}, etc.) that have:

- "Arithmetic" operator $\oplus \sim +$
- "Multiplicative" operator $\otimes \sim *$
- A *zero* member
- Negatives and Reciprocals

Field operations always produce another element in the set. (Closure)

In this discussion, our Galois Field has:

- 256 elements (*thingees*) <u>labeled</u> 0x01 to 0xFF, and 0x00 - [*GF(8)*]
- An "Arithmetic" operator (addition and subtraction) ~ XOR $\oplus$
- A "Multiplicative" operator (* and ÷) ~ exponent operations $\otimes$

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Galois "Multiplicative" Operator

$$(X)\otimes(Y) = (\alpha^{\log_\alpha X})\otimes(\alpha^{\log_\alpha Y}) = \alpha^{[(\log_\alpha X + \log_\alpha Y) \text{ modulus } 255]}$$

255 non-zero members

Noting that $\alpha^K \otimes \alpha^{-K} = \alpha^{K-K} = \alpha^0 = 1$

...it follows that $1/(\alpha^K) = \alpha^{-K}$

So...

$$(X)/(Y) = (\alpha^{\log_\alpha X})/(\alpha^{\log_\alpha Y})$$
$$= (\alpha^{\log_\alpha X})\otimes(1/(\alpha^{\log_\alpha Y}))$$
$$= (\alpha^{\log_\alpha X})\otimes(\alpha^{-\log_\alpha Y})$$
$$= \alpha^{[(\log_\alpha X - \log_\alpha Y) \text{ modulus } 255]}$$

Note: Exponent additions/subtractions operations are base 16.

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Example Calculations

$0x2B \otimes 0x6F = 0x8F$

<u>From Log Table (Table 2)</u>

$0x2B = \alpha^{0xDA_{16}} = \alpha^{218_{10}}$ (Base 10 for Understanding Ease)

$0x6F = \alpha^{0x3D_{16}} = \alpha^{61_{10}}$

<u>So</u>

$$0x2B \otimes 0x6F = \alpha^{218_{10}} \otimes \alpha^{61_{10}}$$
$$= \alpha^{(218_{10} + 61_{10})\bmod 255_{10}}$$
$$= \alpha^{(279_{10})\bmod 255_{10}}$$
$$= \alpha^{24_{10}}$$
$$= \alpha^{0x18_{16}}$$

<u>From Powers Table (Table 1)</u>

$\alpha^{0x18} = 0x8F$

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Example Calculations (cont.)

Arithmetic (Addition, Subtraction):

$0x2D \oplus 0x28 == 0010.1101_b$ XOR

$0010.1000_b = 0000.0101_b = 0x05$

Multiplication: $0x3F \otimes 0x12$

$= \alpha^{0xA6} \otimes \alpha^{0xE0}$ (From Table 2)

$= \alpha^{0xA6 + 0xE0}$ (Base 16 Regular Addition)

$= \alpha^{(0x0186) \bmod 255}$

$= \alpha^{0x87}$

$= 0xA9$ (From Table 1)

Division: $0x3F \otimes (1/(0x12))$

$= \alpha^{0xA6} \otimes \alpha^{-0xE0}$ (From Table 2)

$= \alpha^{0xA6 - 0xE0}$ (Base 16 Regular Subtraction)

$= \alpha^{0x01A5 - 0xE0}$ (0xA6 = 0xA6 + 0xFF = ox01A5)

$= \alpha^{0xC5} = 0x8D$ (From Table 1)

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Oddities

Negative Values

$\oplus \approx$ XOR

Thus, for any element V, V = -V since (V XOR V) = 0

Reciprocals

Let $\log_\alpha V == 0xNN$ ($V \neq 0$)

Then $1/V = 1/(\alpha^{0xNN}) = \alpha^{-0xNN}$

Proof: $v/v = 1 = (\alpha^{0xNN}) / (\alpha^{0xNN}) = (\alpha^{0xNN}) \otimes [1/(\alpha^{0xNN})]$

$= (\alpha^{0xNN}) \otimes (\alpha^{-0xNN})$ (?)

$= (\alpha^{0xNN + -0xNN})$

$= \alpha^0$

$= 1$

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Double Data Disk Failure

Remembering

Parity Disk P Generation: $D_1 \oplus D_2 \oplus D_3 \oplus \ldots \oplus D_{N-2} = P$ (Normal RAID 5 Parity Generation)

Parity Disk Q Generation: $(K_1 \otimes D_1) \oplus (K_2 \otimes D_2) \oplus (K_3 \otimes D_3) \oplus \ldots \oplus (K_{N-2} \otimes D_{N-2}) = Q$

Losing Data Disk 1 and 2 gives

$D_1 \oplus D_2 = P \oplus D_3 \oplus \ldots \oplus D_{N-2} = V_1$ (1)

$(K_1 \otimes D_1) \oplus (K_2 \otimes D_2) = Q \oplus (K_3 \otimes D_3) \oplus \ldots (K_{N-2} \otimes D_{N-2}) = V_2$ (2)

So … $(K_1 \otimes D_1) \oplus (K_1 \otimes D_2) = (K_1 \otimes V_1)$

$\oplus$ $(K_1 \otimes D_1) \oplus (K_2 \otimes D_2) = V_2$

Gives … $D_2 \otimes (K_1 \oplus K_2) = (K_1 \otimes V_1) \oplus V_2$

Or … $D_2 = ((K_1 \otimes V_1) \oplus V_2) / (K_1 \oplus K_2)$

Recover D_1 using RAID 5 Logic with P values…

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Summary

Easy to Solve Independent Linear Equations

$$X + Y = 3$$

$$3*X - Y = 1$$

Similarly, Losing Two Data Values Results in:

$$X \oplus Y = V_1$$

$$0x3 \otimes X \oplus Y = V_2$$

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The End

Thank you!

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