Quasi-Cyclic Non-Binary LDPC Codes for MLC NAND Flash Memory

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Joint work with:
Clayton Schoeny (UCLA),
Behzad Amiri (UCLA), and
Lara Dolecek (UCLA)

Flash Memory Summit 2015

1. MLC NAND Flash Model
   - Mixed Normal-Laplace Distribution
   - Capacity

C. Schoeny, B. Amiri, A. Hareedy, and L. Dolecek, "Quasi-Cyclic Non-Binary LDPC Codes for MLC NAND Flash Memory", NVMW, March 2015.

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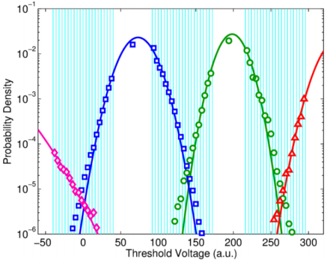
Flash Model

Normal-Laplace Distribution

Key properties of the *Normal-Laplace* Distribution:

- Normal-like curve but wider tails.
- The two tails can behave differently from one another.



Normal

Level 0
Level 1
Level 2
Level 3

T. Parnell, N. Papandreou, T. Mittelholzer, and H. Pozidis, "Modelling of the Threshold Voltage Distributions of Sub-20nm NAND Flash Memory", Globecomm, Dec. 2014.

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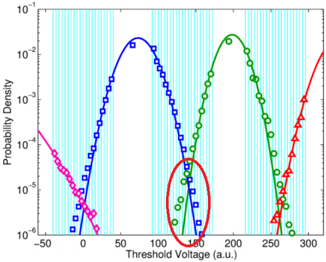

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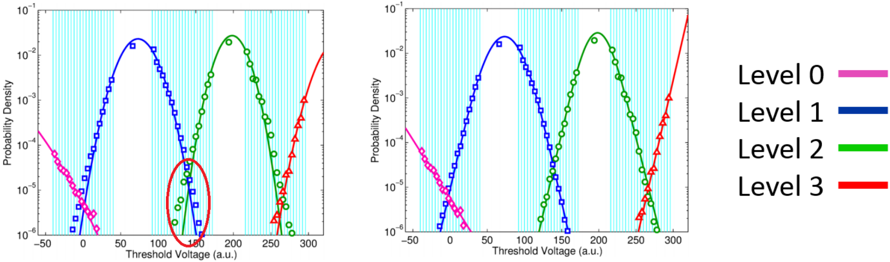

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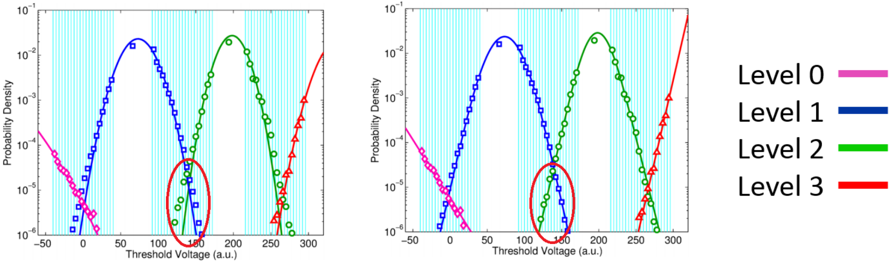

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Multi-modality is caused by *programming errors*, in which we program the wrong level.

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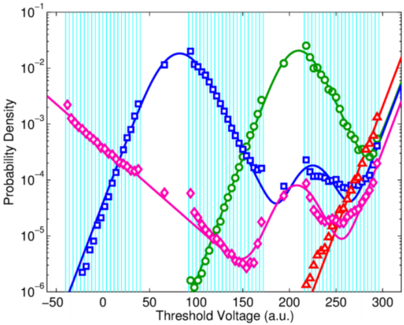
- Error in two step programming algorithm.



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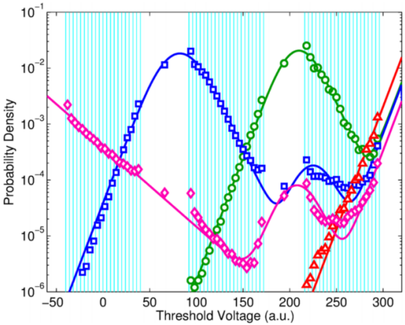

The NL-distributions and the programming error rates are parameterized by the P/E cycles.

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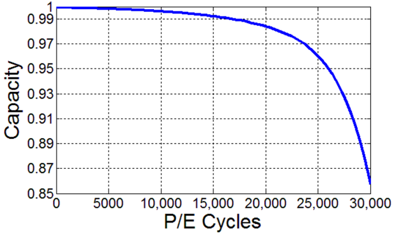
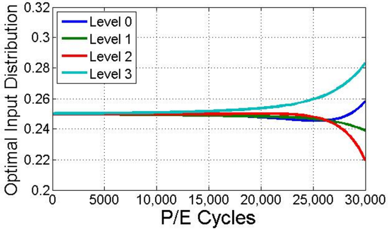
Capacity and Optimal Input Distribution





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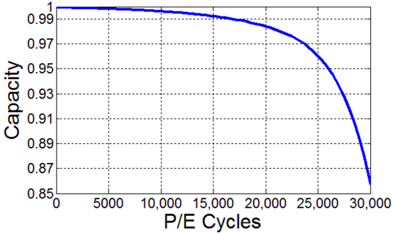

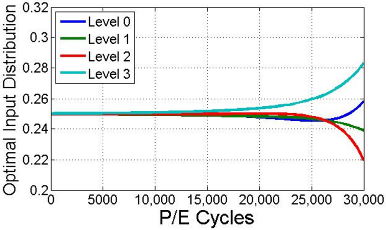

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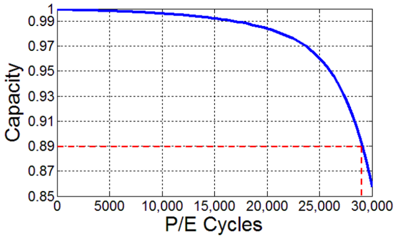
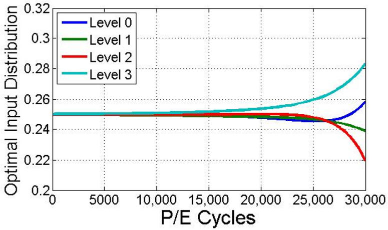
Capacity and Optimal Input Distribution - Longer Lifetime is Possible





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- Optimal input distribution are uniform until late in lifetime.
- Codes with rate 8/9 can support up to 28,000 P/E cycles.

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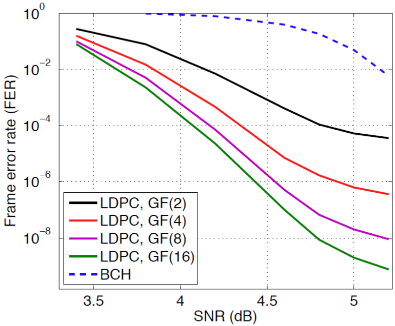
LDPC Codes

Why Non-Binary LDPC Codes?



Regular codes
Blocklength $N = 1000$ bits
Rate $R = 0.9$
Column-weight $\ell = 4$

- LDPC codes outperform commonly used BCH codes.
- Larger Galois field size results in better performance.

Binary LDPC Codes

LDPC codes are a class of graph-based channel codes with capacity approaching performance. These codes can be described by a bipartite graph called a Tanner graph.

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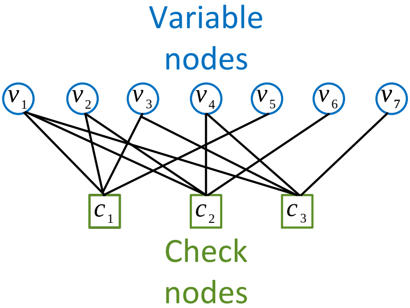
Binary code

$$
\begin{array}{cccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\
\end{array}
$$

$$
H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix}
$$



Parity check: $c_1 = v_1 + v_2 + v_3 + v_5$ over $GF(2)$.

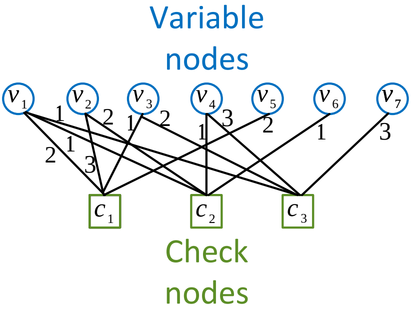
Non-Binary LDPC Codes

LDPC codes are a class of graph-based channel codes with capacity approaching performance. These codes can be described by a bipartite graph called a Tanner graph.

Non-binary code, GF(4)

$$H = \begin{array}{c} \\ \end{array}\begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array}$$

$$H = \begin{bmatrix} 2 & 3 & 1 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 3 \end{bmatrix} \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix}$$



Parity check: $c_1 = 2v_1 + 3v_2 + v_3 + 2v_5$ over $GF(4)$.

Non-Binary LDPC Decoding Complexity

Commonly used decoding algorithms:

- Binary: Min-Sum
- Non-Binary: Min-Max

D. Declercq and M. Fossorier, "Decoding Algorithms for Nonbinary LDPC Codes Over GF", TCOM, Apr. 2007.

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Current research includes complexity reduction through the use of message pruning.

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Quasi-Cyclic Non-Binary Advantages

- Construction is based on lifting and labeling protographs.

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- Construction is based on lifting and labeling protographs.
- Due to their implementation-friendly structure and superior performance, QC-NB codes are well-suited for emerging data storage applications requiring very low error rates.

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We design QC-NB codes with improved performance in the low error-rate region by removing small problematic absorbing sets while maintaining desired code parameters.

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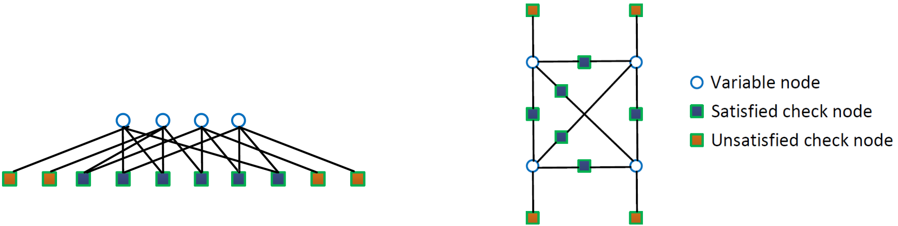
Error Floor is Caused by Absorbing Sets

LDPC codes suffer from an error floor when decoded by message passing algorithms, largely due to subgraphs called absorbing sets.

Example: A (4,4) binary absorbing set.



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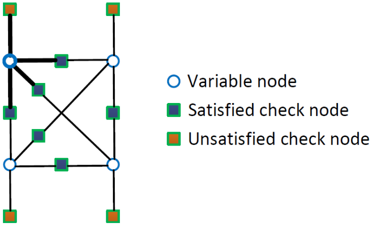

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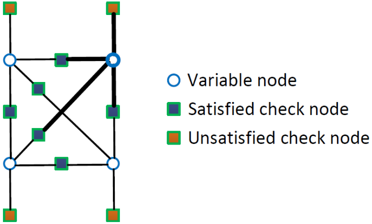

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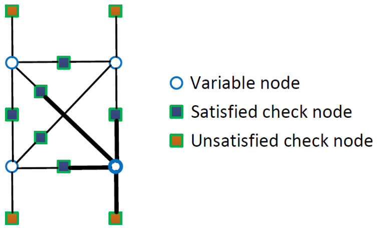

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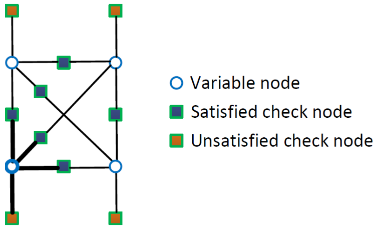

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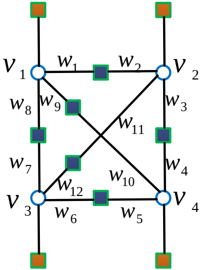
Non-Binary Absorbing Sets

- Edge weights and variable node values are elements of $GF(q)$.
- The shown absorbing set is called elementary absorbing set.



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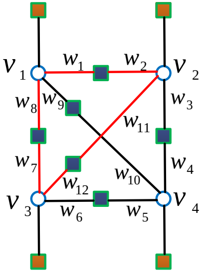

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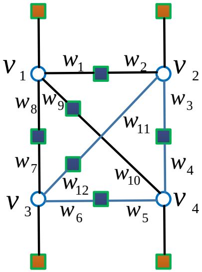

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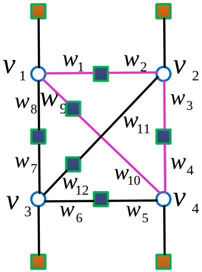

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Absorbing Set Removal

1. Identify a list of problematic absorbing sets.

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3. For each candidate, check if the weight conditions are satisfied.
4. The labeling parameters in the process of constructing the QC-NB-LDPC code are modified such that the weight condition of absorbing sets is not satisfied.
5. Continue until no more non-binary absorbing sets can be eliminated.

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Simulation Results

Shifting Gaussian MLC Channel

First let us view the results from a previously used model.

Each state is modeled as a Gaussian distribution with shifting mean and variance dependent on the P/E cycles.

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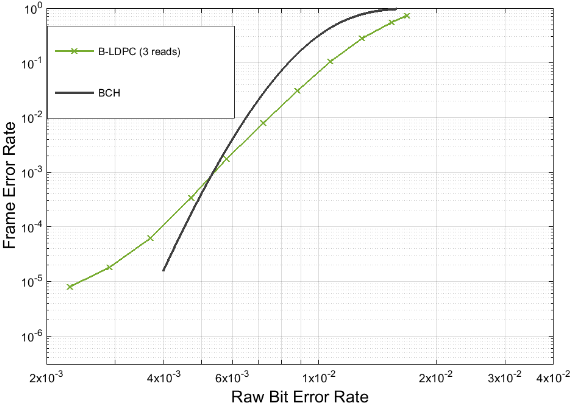
Each state is modeled as a Gaussian distribution with shifting mean and variance dependent on the P/E cycles.

- Binary LDPC code decoded using the min-sum algorithm.
- Non-binary LDPC code decoded using FFT-sum-product algorithm.
- Block lengths $\approx 2k$ bits.
- Code rates $\approx 8/9$.
- GF sizes $= 4$.
- Column weights $= 4$.

Shifting Gaussian MLC Channel



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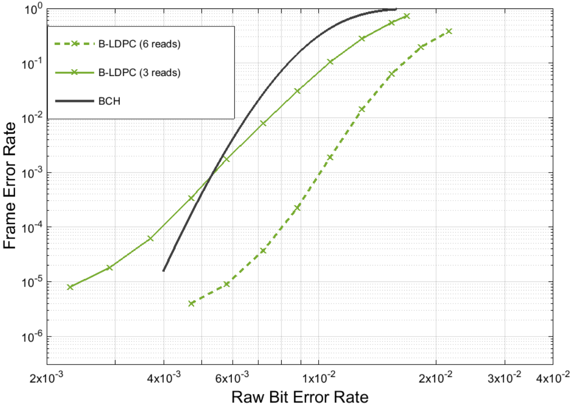

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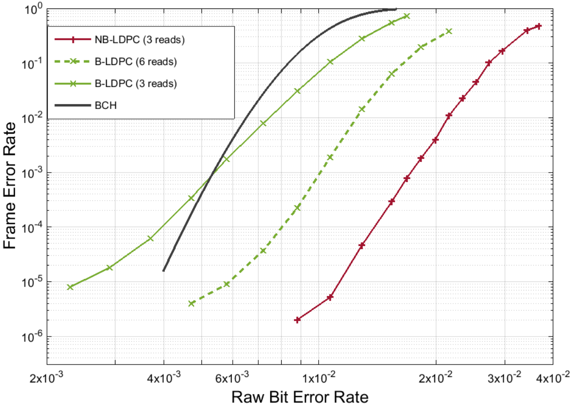

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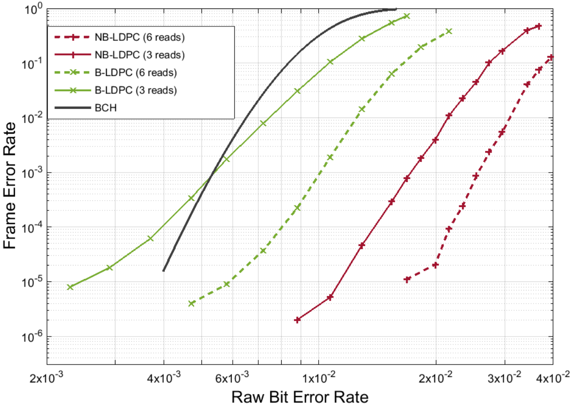

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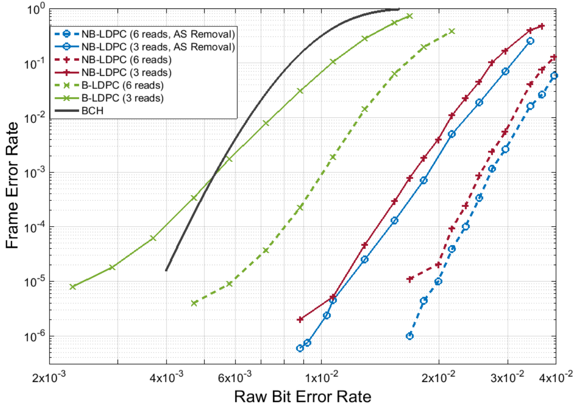
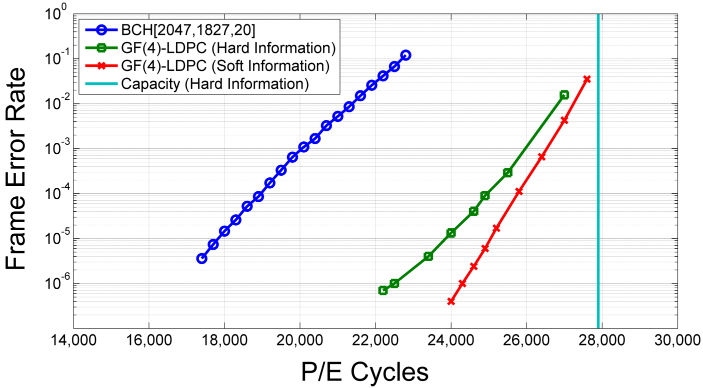


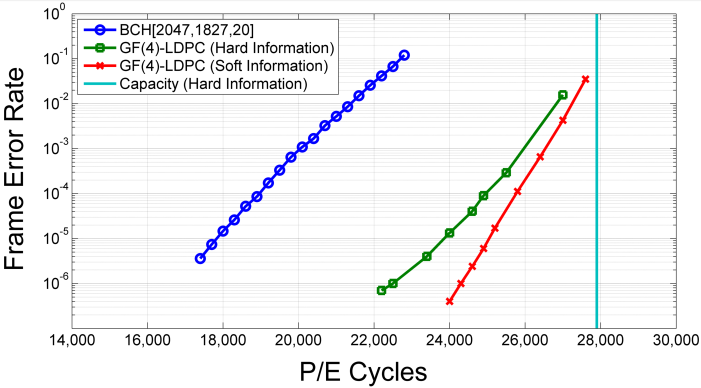
Superior performance for non-binary LDPC codes with AS removal compared to BCH and binary LDPC codes.

Normal-Laplace Mixture MLC Channel



- ”Soft-Information” refers to 6-reads.

Normal-Laplace Mixture MLC Channel



- Block length = 1692 bits.
- Code rate $\approx 8/9$.
- GF size = 4.
- Column weight = 4.

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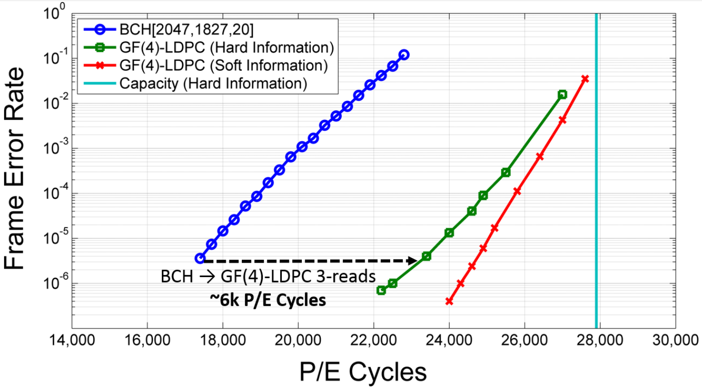

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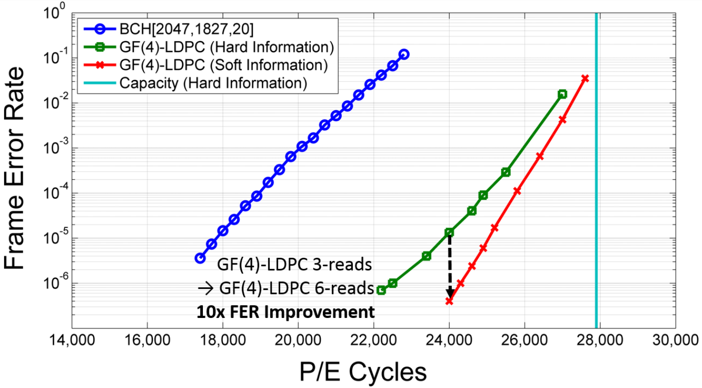

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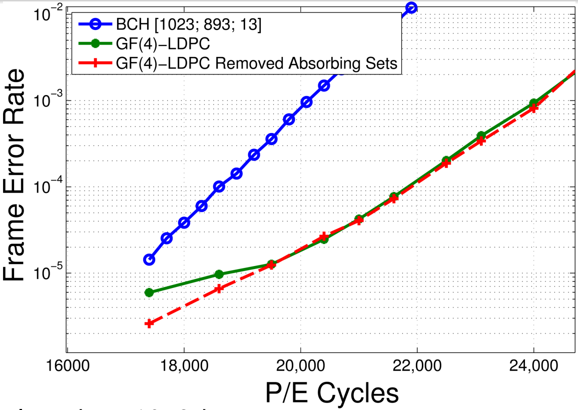
- Column weight = 4.

Normal-Laplace Mixture MLC Channel



- Block length = 1058 bits.
- Code rate ≈ 8/9.
- GF size = 4.
- Column weight = 3.

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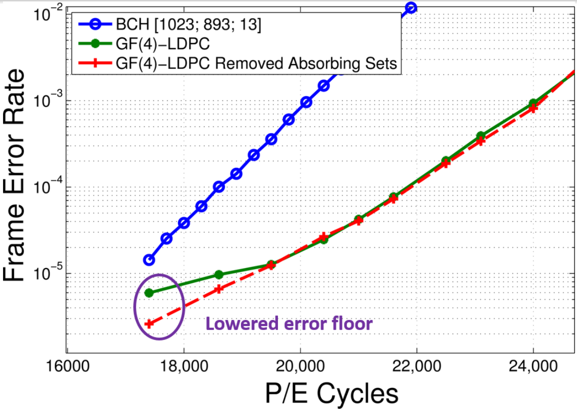

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- Non-binary LDPC codes offer excellent error-correcting performance over NAND Flash models.
- AS removal provides very promising results over the Gaussian Flash channel model.
- Further performance improvement is achievable over the NL model by accurately identifying the objects which dominate the error floor (ongoing research).
- Concurrent work on partial-response channels demonstrated significant performance gain by properly identifying and removing the right objects.

A. Hareedy, B. Amiri, S. Zhao, R. Galbraith, and L. Dolecek, "Non-Binary LDPC Code Optimization for Partial-Response Channels", Globecom, Dec. 2015.

Thank You